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# Toplam Sembolü:

$$\sum_{k=1}^{n} a_k = a_1 + a_2 + a_3 + \dots + a_n$$

**Kural:** $\sum_{k=1}^{n} k = 1+2+3+\dots+n = \frac{n.(n+1)}{2}$

**örnek:** $\sum_{k=1}^{5} k = \frac{5.6}{2} = 15$

**Kural:** $\sum_{k=p}^{n} c\,a_k = c.\sum_{k=p}^{n} a_k$

**örnek:** $\sum_{k=1}^{5} 4k = 4\sum_{k=1}^{5} k = 4.\frac{5.6}{2} = 60$

**Kural:** $\sum_{k=1}^{n} c = \underbrace{c+c+\dots+c}_{n\ tane} = n.c$

**örnek:** $\sum_{k=1}^{7} 4 = 7.4 = 28$

**Kural:** $\sum_{k=1}^{n} (a_k+b_k-c_k) = \sum_{k=1}^{n} a_k + \sum_{k=1}^{n} b_k - \sum_{k=1}^{n} c_k$

**örnek:** $\sum_{k=1}^{10}(10k+3) = 10.\sum_{k=1}^{10} k + \sum_{k=1}^{10} 3 = 10.\frac{10.11}{2} + 10.3$

$= 580$

**Kural:** $\sum_{k=p}^{n} a_k = \sum_{k=p-r}^{n-r} a_{k+r}$

$\sum_{k=p}^{n} a_k = \sum_{k=p+r}^{n+r} a_{k-r}$

**örnek:** $\sum_{k=5}^{12} k = \sum_{k=5-4}^{12-4} (k+4) = \sum_{k=1}^{8} k+4$

$= \sum_{k=1}^{8} k + \sum_{k=1}^{8} 4 = \frac{8.9}{2} + 8.4 = 68$

**örnek:** $\sum_{k=-4}^{5} (2k-5) = \sum_{k=-4+5}^{5+5} [2(k-5)-5]$

$= \sum_{k=1}^{10} (2k-15) = 2\sum_{k=1}^{10} k - \sum_{k=1}^{10} 15$

$= 2.\frac{10.11}{2} - 10.15 = 110-150 = -40$

**Kural:** $\sum_{k=1}^{n} k^2 = 1^2+2^2+\dots+n^2 = \frac{n.(n+1).(2n+1)}{6}$

**örnek:** $\sum_{k=3}^{7} k^2 = \sum_{k=3-2}^{7-2} (k+2)^2 = \sum_{k=1}^{5} k^2+4k+4$

$= \sum_{k=1}^{5} k^2 + 4\sum_{k=1}^{5} k + \sum_{k=1}^{5} 4 = \frac{5.6.11}{6} + 4.\frac{5.6}{2} + 5.4$

$= 55+60+20 = 135$

**Kural:** $\sum_{k=1}^{n} k^3 = 1^3+2^3+\dots+n^3 = \left[\frac{n.(n+1)}{2}\right]^2$

**örnek:** $\sum_{k=1}^{4} k^3 = \left[\frac{4.5}{2}\right]^2 = 100$

**Kural:** $\sum_{k=0}^{n} r^k = 1+r+r^2+\dots+r^n = \frac{r^{n+1}-1}{r-1}$

**örnek:** $\sum_{k=0}^{20} 2^k = 1+2+2^2+\dots+2^{20} = \frac{2^{21}-1}{2-1} = 2^{21}-1$

**Kural:** $\sum_{k=1}^{n} k.k! = 1.1!+2.2!+\dots+n.n! = (n+1)!-1$

**örnek:** $\sum_{k=3}^{14} k.k! = 3.3!+4.4!+\dots+14.14!$

$= \underbrace{(1.1!+2.2!)+3.3!+4.4!+\dots+14.14!}_{(15!-1)} \underbrace{-(1.1!+2.2!)}_{-(3!-1)}$

$= 15!-3!$

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## Basit Kesirlere Ayırma:

$$\frac{1}{k.(k+1)} = \frac{A}{k} + \frac{B}{k+1}, \quad 1 = A(k+1) + Bk$$

$(k+1)$ $(k)$

$k=0$ ise $A=1$

$k=-1$ ise $B=-1$

$$* \quad \frac{1}{k.(k+1)} = \frac{1}{k} - \frac{1}{k+1}$$

**Kural:** Paydadaki çarpanlardan küçük olanı A nın, büyük olanı B nin altına yazılır. Buna göre, çarpanlar arasındaki fark:

→ 1 ise $A=1$, $B=-1$

→ 2 ise $A=\frac{1}{2}$, $B=-\frac{1}{2}$

→ 3 ise $A=\frac{1}{3}$, $B=-\frac{1}{3}$

⋮

**örnek:** $\sum_{k=1}^{7} \frac{1}{k(k+1)} = \sum_{k=1}^{7} \frac{1}{k} - \frac{1}{k+1}$

$$= \begin{matrix} \frac{1}{1} - \frac{1}{2} \\ \frac{1}{2} - \frac{1}{3} \\ \vdots \\ \frac{1}{7} - \frac{1}{8} \end{matrix} \Bigg\} = \frac{1}{1} - \frac{1}{8} = \frac{7}{8}$$

(8)

**örnek:** $\sum_{k=1}^{5} \frac{6}{k(k+2)} = 6\sum_{k=1}^{5} \frac{1}{2}\left(\frac{1}{k} - \frac{1}{k+2}\right)$

$$= 3\underbrace{\sum_{k=1}^{5} \frac{1}{k} - \frac{1}{k+2}}_{\Rightarrow} \quad \begin{matrix} \frac{1}{1} - \frac{1}{3} \\ \frac{1}{2} - \frac{1}{4} \\ \frac{1}{3} - \frac{1}{5} \\ \frac{1}{4} - \frac{1}{6} \\ + \quad \frac{1}{5} - \frac{1}{7} \\ \hline \left(\frac{1}{1} + \frac{1}{2}\right) - \left(\frac{1}{6} + \frac{1}{7}\right) \end{matrix}$$

(2) 7 6

$$= 3.\left(\frac{3}{2} - \frac{13}{42}\right)$$

$$= \frac{9}{2} - \frac{13}{14}$$

(7)

$$= \frac{63-13}{14} = \frac{50}{14} = \frac{25}{7}$$



## Alıştırmalar:

1) $\sum_{k=3}^{14} 3 = \sum_{k=1}^{12} 3 = 3.12 = 36$

2) $\sum_{k=1}^{40} 2n = 40.2n = 80n$

3) $\sum_{k=1}^{n+7} \frac{1983}{n+7} = n+7.\frac{1983}{n+7} = 1983$

4) $\sum_{k=1}^{n} 4n = 64$ ise $\sum_{k=1}^{n+2} (n+3) = ?$

Çözüm! $4n.n = 64$, $n=4$

$$\sum_{k=1}^{n+2} (n+3) = \sum_{k=1}^{6} 7 = 42$$

5) $\sum_{k=-3}^{5} 10 = \sum_{k=1}^{9} 10 = 90$

6) $\sum_{k=1}^{10} 4k = 4\sum_{k=1}^{10} k = 4.\frac{10.11}{2} = 220$

7) $\sum_{k=1}^{8} 2k^2 = 2\sum_{k=1}^{8} k^2 = 2.\frac{8.9.17}{6} = 408$

8) $\sum_{n=1}^{20} (n+2) = \sum_{n=1}^{20} n + \sum_{n=1}^{20} 2 = \frac{20.21}{2} + 20.2$

$= 250$

9) $\sum_{n=1}^{20} (2+na) = 70$ ise $a = ?$

Çözüm! $\sum_{n=1}^{20} 2 + a.\sum_{n=1}^{20} n = 70$

$20.2 + a.\frac{20.21}{2} = 70$, $a.10.21 = 30$

$a = \frac{1}{7}$

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**10)** $\sum_{k=1}^{5}(k^2-3k+6)=?$

**Çözüm:** $\sum_{k=1}^{5}k^2-3\sum_{k=1}^{5}k+\sum_{k=1}^{5}6$

$=\frac{5.6.11}{6}-3.\frac{5.6}{2}+5.6=55-45+30=40$

**11)** $\sum_{k=1}^{10}k.(k+1)=?$

**Çözüm:** $\sum_{k=1}^{10}k^2+k=\sum_{k=1}^{10}k^2+\sum_{k=1}^{10}k$

$=\frac{10.11.21}{6}+\frac{10.11}{2}=385+55=440$

**12)** $\sum_{k=1}^{12}(k+2).(k-1)=?$

**Çözüm:** $\sum_{k=1}^{12}k^2+k-2=\sum_{k=1}^{12}k^2+\sum_{k=1}^{12}k-\sum_{k=1}^{12}2$

$=\frac{12.13.25}{6}+\frac{12.13}{2}-12.2=650+78-24=704$

**13)** $\sum_{k=1}^{20}2^k=?$

**Çözüm:** $2+2^2+\dots+2^{20}=2(1+2+\dots+2^{19})$

$=2.\frac{2^{20}-1}{2-1}=2^{21}-2$

**14)** $\sum_{k=1}^{24}3^{k+2}=?$

**Çözüm:** $3^3+3^4+\dots+3^{26}=3^3(1+3+3^2+\dots+3^{23})$

$=27.\frac{3^{24}-1}{3-1}=\frac{27}{2}.(3^{24}-1)$

**15)** $\sum_{k=3}^{15}(k-2)=?$

**Çözüm:** $\sum_{k=1}^{13}k=\frac{13.14}{2}=91$

**16)** $\sum_{k=-4}^{7}(2k+6)=?$

**Çözüm:** $\sum_{k=1}^{12}(2(k-5)+6)=\sum_{k=1}^{12}2k-4$

$=2\sum_{k=1}^{12}k-\sum_{k=1}^{12}4=2.\frac{12.13}{2}-12.4=156-48$

$=108$

**17)** $\sum_{k=5}^{13}(4k+2)=?$

**Çözüm:** $\sum_{k=1}^{9}(4(k+4)+2)=\sum_{k=1}^{9}(4k+18)$

$=4\sum_{k=1}^{9}k+\sum_{k=1}^{9}18=4.\frac{9.10}{2}+9.18=180+162$

$=342$

**18)** $a_n=\sum_{k=1}^{n}\frac{1}{k.(k+1)}$ ise $a_{99}=?$

**Çözüm:** $a_{99}=\sum_{k=1}^{99}\frac{1}{k}-\frac{1}{k+1}=\frac{1}{1}-\frac{1}{100}=\frac{99}{100}$

**19)** $\sum_{k=1}^{20}\frac{1}{k^2+3k+2}=?$

**Çözüm:** $\sum_{k=1}^{20}\frac{1}{(k+1)(k+2)}=\sum_{k=1}^{20}\frac{1}{k+1}-\frac{1}{k+2}$

$=\frac{1}{1+1}-\frac{1}{20+2}=\frac{1}{2}-\frac{1}{22}=\frac{10}{22}=\frac{5}{11}$

**20)** $\sum_{k=4}^{50}\frac{1}{k^2-5k+6}=?$

**Çözüm:** $\sum_{k=4}^{50}\frac{1}{(k-3)(k-2)}=\sum_{k=4}^{50}\frac{1}{k-3}-\frac{1}{k-2}$

$=\frac{1}{4-3}-\frac{1}{50-2}=\frac{1}{1}-\frac{1}{48}=\frac{47}{48}$

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**21)** $\sum_{k=3}^{20} (k+1).(k+1)! = ?$

**Çözüm!** $4.4! + 5.5! + \dots + 21.21!$

$= \underbrace{(1.1! + 2.2! + 3.3!) + 4.4! + 5.5! + \dots + 21.21!}_{(22!-1)} - \underbrace{(1.1! + \dots + 3.3!)}_{(4!-1)}$

$= 22! - 4!$

**22)** $\sum_{p=1}^{n} p.(p+1) = \frac{n.(n^2+an+b)}{3}$ ise $2a+b = ?$

**Çözüm!** $\sum_{p=1}^{n} p^2+p = \sum_{p=1}^{n} p^2 + \sum_{p=1}^{n} p$

$\Rightarrow \frac{n.(n+1).(2n+1)}{6} + \frac{n.(n+1)}{2} = \frac{n.(n^2+an+b)}{3}$

$\frac{2n^2+3n+1+3n+3}{6} = \frac{2n^2+2an+2b}{6}$

$2n^2+6n+4 = 2n^2+2an+2b$

$a=3,\ b=2\ ,\ 2a+b = 2.3+2 = 8$

**Kural:** $\sum_{k=1}^{n} a_k = \sum_{k=1}^{p} a_k + \sum_{k=p+1}^{n} a_k$

**örnek:** $\sum_{k=1}^{20} a_k = \sum_{k=1}^{8} a_k + \sum_{k=9}^{20} a_k$

**23)** $\sum_{k=1}^{30} a_k = 120$ ve $\sum_{k=1}^{12} a_k = 24$ ise $\sum_{k=13}^{30} a_k = ?$

**Çözüm!**

$\underbrace{\sum_{k=1}^{30} a_k}_{120} = \underbrace{\sum_{k=1}^{12} a_k}_{24} + \underbrace{\sum_{k=13}^{30} a_k}_{x}$ , $x = 96$

**24)** $\sum_{k=1}^{48} (\sqrt{k+1} - \sqrt{k}) = ?$

**Çözüm!**

$\sqrt{2}-\sqrt{1}$
$\sqrt{3}-\sqrt{2}$
$\sqrt{4}-\sqrt{3}$
$\vdots$
$\sqrt{49}-\sqrt{48}$

$\sqrt{49}-\sqrt{1} = 7-1 = 6$

**25)** $\sum_{k=1}^{24} (\sqrt{2k+1} - \sqrt{2k-1}) = ?$

**Çözüm!**

$\sqrt{3}-\sqrt{1}$
$\sqrt{5}-\sqrt{3}$
$\sqrt{7}-\sqrt{5}$
$\vdots$
$\sqrt{49}-\sqrt{47}$

$\sqrt{49}-\sqrt{1} = 7-1 = 6$

**26)** $x^2-3x-1=0$ denkleminin kökleri $x_1$ ve $x_2$ dir. Buna göre, $\sum_{k=1}^{2} 5x_k = ?$

**Çözüm!** $x^2-3x-1=0$, $x_1+x_2 = -\frac{b}{a}$, $x_1+x_2 = 3$

$\sum_{k=1}^{2} 5x_k = 5x_1 + 5x_2 = 5(x_1+x_2) = 15$

**27)** $\sum_{p=0}^{80} i^p = ?$

**Çözüm!** $\underbrace{i^0 + i^1 + i^2 + \dots + i^{80}}_{0} = i^0 = 1$

**28)** $\sum_{n=1}^{4} \sum_{m=2}^{3} (m^2n - 6n) = ?$

**Çözüm:** $\sum_{n=1}^{4} \underbrace{(4n-6n)}_{-2n} + \underbrace{(9n-6n)}_{3n} = \sum_{n=1}^{4} n$

$= \frac{4.5}{2} = 10$

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**29)** $\sum_{k=1}^{4}\sum_{s=1}^{2}(4s-2k+1)=?$

**Çözüm:** $\sum_{k=1}^{4}(s-2k)+(9-2k)=\sum_{k=1}^{4}14-4k$

$=\sum_{k=1}^{4}14-4\sum_{k=1}^{4}k=14.4-4.\frac{4.5}{2}=56-40=16$

**30)** $\sum_{j=1}^{4}\sum_{i=0}^{3}(3i-2j+1)=?$

**Çözüm:** $\sum_{j=1}^{4}(-2j+1)+(4-2j)+(7-2j)+(10-2j)$

$=\sum_{j=1}^{4}22-8j=\sum_{j=1}^{4}22-8\sum_{j=1}^{4}j=4.22-8.\frac{4.5}{2}$

$=88-80=8$

**31)** $f(x)=3x+1$, $x_1=1$, $x_2=4$ ise

$\sum_{i=1}^{2}(x_i-3).f(x_i)=?$

**Çözüm:** $(x_1-3).f(x_1)+(x_2-3).f(x_2)$

$\downarrow 1 \quad \downarrow 1 \quad \downarrow 4 \quad \downarrow 4$

$-2.f(1)+1.f(4)=-8+13=5$

**32)** $\sum_{k=1}^{30}(-1)^k.(k+1)=?$

**Çözüm:** $\underbrace{-2+3}_{1}\underbrace{-4+5}_{1}\dots\underbrace{-30+31}_{1}=15$

**33)** $\sum_{k=1}^{45}(-1)^k.(k+1)=?$

**Çözüm:** $\overbrace{-2+3}^{1}\overbrace{-4+5}^{1}\dots\overbrace{-44+45}^{1}-46$

$22 \qquad -46=-24$

**34)** $\sum_{p=1}^{40}\log_3\left(\frac{p+5}{p+4}\right)=?$

**Çözüm:** $\log_3\left(\frac{6}{5}\right)+\log_3\left(\frac{7}{6}\right)+\dots+\log_3\left(\frac{45}{44}\right)$

$=\log_3\left(\frac{6}{5}.\frac{7}{6}.\frac{8}{7}\dots\frac{45}{44}\right)=\log_3\left(\frac{45}{5}\right)=\log_3 9=2$

**35)** $\sum_{k=1}^{90}\cos^2 k=?$

**Çözüm:**

$\cos^2 1+\cos^2 2+\cos^2 3+\dots+\cos^2 88+\cos^2 89+\cos^2 90$

$=\cos^2 1+\cos^2 2+\dots+\cos^2 44+\cos^2 45+\sin^2 44+\dots+\sin^2 1+0$

$=\underbrace{1+1+1+\dots+1}_{44 \text{ tane}}+\cos^2 45+0=44+\frac{1}{2}=\frac{89}{2}$

## $S_n$: ilk n terimin toplamı:

$S_n=\sum_{k=1}^{n}a_k=a_1+a_2+\dots+a_n$

$S_1=a_1$

$S_2=a_1+a_2$

$S_3=a_1+a_2+a_3$

$\vdots$

$S_n=S_{n-1}+a_n \rightarrow \boxed{a_n=S_n-S_{n-1}}$

$a_7=S_7-S_6$

$a_5=S_5-S_4$

**36)** ilk n terimin toplamı $S_n=\sum_{k=1}^{n}a_k=n^2+2n+10$ olan dizinin 6. terimi kaçtır?

**Çözüm:**

$a_6=S_6-S_5$

$=(6^2+6.2+10)-(5^2+2.5+10)$

$=58-45$

$=13$

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# Çarpım Sembolü:

$$\prod_{k=1}^{n} a_k = a_1 . a_2 \dots a_n$$

**Kural:**

$$\prod_{k=1}^{n} k = 1.2.3 \dots n = n!$$

$$\prod_{k=1}^{8} k = 8!$$

**Örnek:** $\prod_{k=7}^{20} k = 7.8.9 \dots 20 = \frac{1.2 \dots 6.7.8 \dots 20}{1.2 \dots 6}$

$$= \frac{20!}{6!}$$

**Kural:**

$$\prod_{k=1}^{n} a = \underbrace{a.a \dots a}_{n \text{ tane}} = a^n$$

$$\prod_{k=1}^{10} 3 = 3^{10}$$

**Kural:**

$$\prod_{k=1}^{n} a.k = a^n . \prod_{k=1}^{n} k$$

$$\prod_{k=1}^{12} 3k = 3^{12} \prod_{k=1}^{12} k = 3^{12} . 12!$$

**Örnek:** $\prod_{k=5}^{10} 4 = \prod_{k=5-4}^{10-4} 4 = \prod_{k=1}^{6} 4 = 4^6$

**37)** $\prod_{n=5}^{12} \sum_{k=1}^{n} \frac{1}{k.(k+1)} = ?$

**Çözüm:** $\prod_{n=5}^{12} \frac{n}{n+1} = \frac{5}{6} . \frac{6}{7} . \frac{7}{8} \dots \frac{12}{13} = \frac{5}{13}$

**38)** $\sum_{n=1}^{10} \prod_{m=2}^{8} (mn - 3n) = ?$

**Çözüm:** $\sum_{n=1}^{10} \underbrace{\prod_{m=2}^{8} (mn-3n)}_{(-n).0 \dots = 0} = \sum_{n=1}^{10} 0 = 0$

**39)** $\prod_{k=1}^{45} (k^2 - 2k - 15) = ?$

**Çözüm:** $\prod_{k=1}^{45} (k-5).(k+3) = 0. \dots = 0$

**40)** $\prod_{k=2}^{3} \prod_{n=1}^{4} k = ?$

**Çözüm:** $\prod_{k=2}^{3} k^4 = 2^4 . 3^4 = 6^4$

**41)** $\prod_{k=1}^{n} 2^k = 2^{21}$ ise n kaçtır?

**Çözüm:** $2^1 . 2^2 \dots 2^n = 2^{21}$

$2^{1+2+\dots+n} = 2^{21}$, $\frac{n(n+1)}{2} = 21$, $n(n+1) = 42$

$n = 6$

~~$n = -7$~~

**42)** $\prod_{k=50}^{130} \cos k = ?$

**Çözüm:**

$\cos 50 . \cos 51 \dots \underbrace{\cos 90}_{0} \dots \cos 130 = 0$

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**43)** $\prod_{k=2}^{60} \frac{k^2+2k-3}{k^2+3k-4} = ?$

**Çözüm:**

$$\prod_{k=2}^{60} \frac{(k+3).(k-1)}{(k+4).(k-1)} = \frac{5}{6}.\frac{6}{7}.\frac{7}{8}\cdots\frac{63}{64} = \frac{5}{64}$$

**44)** $\prod_{k=4}^{40} \left(1+\frac{2k+1}{k^2}\right) = ?$

**Çözüm:**

$$\prod_{k=4}^{40} \frac{(k+1)^2}{k^2} = \frac{5^2}{4^2}.\frac{6^2}{5^2}.\frac{7^2}{6^2}\cdots\frac{41^2}{40^2} = \frac{1681}{16}$$

**45)** $\prod_{k=4}^{30} \left(1-\frac{1}{k^2}\right) = ?$

**Çözüm:**

$$\prod_{k=4}^{30} \frac{k^2-1}{k^2} = \prod_{k=4}^{30} \frac{(k-1)}{k}.\frac{(k+1)}{k}$$

$$= \left(\frac{3}{4}.\frac{4}{5}.\frac{5}{6}\cdots\frac{29}{30}\right).\left(\frac{5}{4}.\frac{6}{5}.\frac{7}{6}\cdots\frac{31}{30}\right)$$

$$= \frac{3}{30_{10}}.\frac{31}{4} = \frac{31}{40}$$

**46)** $\prod_{k=-7}^{103} \log_{(k+12)}(k+8) = ?$

**Çözüm:** $\underbrace{\log_5 1}_{0}\ldots\ldots = 0$

**47)** $\prod_{p=2}^{63} \log_p(p+1) = ?$

**Çözüm:**

$$\prod_{p=2}^{63} \frac{\log(p+1)}{\log p} = \frac{\log 3}{\log 2}.\frac{\log 4}{\log 3}\cdots\frac{\log 64}{\log 63}$$

$$= \frac{\log 64}{\log 2} = \frac{\log 2^6}{\log 2} = \frac{6\log 2}{\log 2} = 6$$

**48)** $\prod_{a=1}^{30} 3.a^4 = ?$

**Çözüm:**

$$3^{30}.\prod_{a=1}^{30} a^4 = 3^{30}.1^4.2^4.3^4\ldots 30^4$$

$$= 3^{30}.(1.2.3\ldots 30)^4 = 3^{30}.(30!)^4$$

**49)** $\prod_{k=1}^{50} k^3.3^k = ?$

**Çözüm:**

$$\prod_{k=1}^{50} k^3.3^k = 1^3.2^3\ldots 50^3.3^1.3^2.3^3\ldots 3^{50}$$

$$= (50!)^3.3^{\frac{50.51}{2}} = (50!)^3.3^{1275}$$

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